Faculty of Science
Department of Mathematics

Study Plan for
Master Degree in Mathematics
Thesis Track

2017

Study Plan for the Master of Science Degree in Mathematics
Thesis Track

First: **The applicant to this program is required:**

1. To hold a bachelor's degree in mathematics.
2. To pass the foreign language requirements in accordance with the decisions of the Higher Education Council.

Second: **The Master of Science Degree in Mathematics is awarded upon the fulfillment of the following requirements:**

1. Achievements of the conditions specified in the regulations for awarding the master degree at Yarmouk University No. (3) for the year 2011.
2. Completion of the remedial courses recommended by the Department Graduate Studies Committee.
3. Successfully completing at least (24 credit hours) of the courses of 600 level distributed as follows:

   a) **Obligatory Courses**: (15 credit hrs.) as in *Table (1)* below:

Table (1) **Obligatory Courses (15 credit hrs.) (Thesis Track)**

No.	Course No.	Course Title	Credit Hours	Prerequisite	The semester the course is offered
1.	MATH 601	Theory of Ordinary Differential Equations and its Applications I.	3	---	2nd
2.	MATH 611	Measure Theory and Integration I	3	---	1st
3.	MATH 621	Advanced Numerical Analysis	3	---	2nd
4.	MATH 641	Modern Algebra I	3	---	1st
5.	MATH 661	Advanced General Topology I	3	---	1st

   b) **Elective Courses** (9 credit hrs.) chosen from the following (*Table (2)*)

Table (2) **Elective Courses (9 credit hrs.) (Thesis Track)**

No.	Course No.	Course Title	Credit Hours	Prerequisites
1.	MATH 603	Partial Differential Equations I	3	---
2.	MATH 612	Functional Analysis I	3	---
3.	MATH 613	Complex Analysis I	3	---
4.	MATH 616	Theory of Operators	3	---
5.	MATH 623	Approximation Theory	3	---
6.	MATH 642	Modern Algebra II	3	MATH 641
7.	MATH 643	Modern Algebra III	3	MATH 641

Table (2) (continued)

8.	MATH 648	Advanced Linear Algebra	3	---
9.	MATH 649	Advanced Number Theory and Cryptography		---
10.	MATH 652	Fuzzy Set Theory and its Applications	3	---
11.	MATH 662	Advanced General Topology II	3	---
12.	MATH 663	Algebraic Topology I (Homotopy Theory)	3	---
13.	MATH 671	Advanced Mathematical Methods I	3	---
14.	MATH 676	Applied Graph Theory	3	---
15.	MATH 697	Selected Topics in Mathematics	3	Successfully completing 9 Cr. Hrs.

4. Preparation, and passing successfully the defense of Thesis (MATH 699) (9 credit hrs.) as in the following (*Table (3)*)

Table (3) Credit Hours for Master Thesis

No.	Course No.	Course Title	Credit Hours
1.	MATH 699A	Thesis	0
2.	MATH 699B	Thesis	3
3.	MATH 699C	Thesis	6
4.	MATH 699D	Thesis	9

***Table (4)* Courses offered by the Mathematics Department for the Master Degree Program**

No.	Course No.	Course Title	Credit Hours	Prerequisites
1.	MATH 601	Theory of Ordinary Differential Equations and its Applications I.	3	---
2.	MATH 603	Partial Differential Equations I	3	---
3.	MATH 611	Measure Theory and Integration I	3	---
4.	MATH 612	Functional Analysis I	3	---
5.	MATH 613	Complex Analysis I	3	---
6.	MATH 616	Theory of Operators	3	---
7.	MATH 621	Advanced Numerical Analysis	3	---
8.	MATH 623	Approximation Theory	3	---
9.	MATH 641	Modern Algebra I	3	---
10.	MATH 642	Modern Algebra II	3	MATH 641
11.	MATH 643	Modern Algebra III	3	MATH 641
12.	MATH 648	Advanced Linear Algebra	3	---
13.	MATH 649	Advanced Number Theory and Cryptography		---
14.	MATH 652	Fuzzy Set Theory and its Applications	3	---
15.	MATH 661	Advanced General Topology I	3	---
16.	MATH 662	Advanced General Topology II	3	---
17.	MATH 663	Algebraic Topology I (Homotopy Theory)	3	---
18.	MATH 671	Advanced Mathematical Methods I	3	---
19.	MATH 676	Applied Graph Theory	3	---
20.	MATH 697	Selected Topics in Mathematics	3	Successfully completing 9 Cr. Hrs.

Department of Mathematics
Graduate Courses Description

MATH 601 -Theory of Ordinary Differential Equations and its Applications I (3 credit hrs.)
Solving first order linear systems ODEs. The fundamental Theorem of first order linear systems. Stability Theory of first order linear systems. Theory of the existence and uniqueness of solutions to first order nonlinear systems of ODEs with respect to parameters and initial conditions. Equilibrium and Linearization of nonlinear systems of ODEs. Stability of nonlinear dynamical systems. Liapunov Method of nonlinear systems. Pre-Predator and coexistence models.

Course Outcomes

Upon successful completion of the course, the student shall be able to:

- Solve linear systems of 1st order ODEs having complex or multiple eigenvalues.
- Apply stability theorems to linear systems of first order ODEs
- Deal with the various cases of linear systems of 1st order ODEs in the plane.
- Find the stable and unstable manifolds of nonlinear systems of 1st order ODEs.
- Prove the stability by using appropriate Liapunov functions.
- Apply linearization and stability to nonlinear systems of 1st order ODEs.

MATH 603 - Partial Differential Equations I (3 credit hrs.)
First Order Partial Differential Equations, Quasi-Linear First Order Partial Differential Equations, The Method of Characteristics, First Order Nonlinear Partial Differential Equations, Nonlinear Reaction Diffusion Phenomena, Asymptotic Methods and Nonlinear Evolutions Equations.

Course Outcomes

Upon successful completion of the course, the student shall be able to:

- Understand how to solve linear, quasi-linear, and nonlinear first order partial differential equations.
- Understand the reaction diffusion process and to study some special solutions of some reaction-diffusion partial differential equations.
- Understand and apply some asymptotic method to approximate a solution for certain partial differential equations.

MATH 611 - Measure Theory and Integration I (3 credit hrs.)
Lebesgue outer measure as a generalization of length of an interval. Lebesgue measurable sets. Borel measurable sets. Characterization of Lebesgue measurable sets. Non-measurable sets. Sets of measure zero. Measurable functions and their properties. Step functions. Characteristic functions. Simple functions. Borel measurable functions. Sequences of functions. Convergence in measure. Lebesgue integral of bounded functions. Comparison of Riemann and Lebesgue integrals. Integral of non-negative measurable functions. General Lebesgue integrals. Improper integrals. Differentiation and integration. Functions of bounded variation. Differentiation of an integral. Absolutely continuous functions. L^p-spaces.

Course Outcomes

Upon successful completion of the course, the student shall be able to:

- Understand the definition of Lebesgue measure and Lebesgue integral as an extension to Riemann integral.

- Understand the definition of uniform convergence, pointwise convergence, convergent in measure for a sequence of measurable functions and relate them to integration.
- Understand the proof and application of some famous theorems, Fatou's lemma, and Lebesgue convergence theorem.
- Understand the definitions of absolutely continuous functions and relate it to differentiable functions, and functions of bounded variation.
- Understand of the structure of L^p-spaces and some of their properties.

MATH 612 - Functional Analysis I **(3 credit hrs.)**

Normed and Banach spaces (completion, product and quotients of normed spaces), Finite dimensional normed spaces and subspaces, Boundedness and continuity of linear functional. The dual spaces. Inner product spaces. Hilbert spaces (orthogonal sets, representation of functionals in Hilbert spaces, Hilbert adjoint operator, self-adjoint unitary and normal operators). Hahn-Banach Theorem. Bounded linear functionals in $C[a, b]$. Reflexive spaces. Uniform boundedness Theorem. Open mapping Theorem. Closed linear operators. Closed graph Theorem. Banach fixed point Theorem and its application to integral equations. Basic concepts in the spectral theory in normed spaces. Spectral properties of bounded linear operators. Spectral mapping theorem for polynomials. Holomorphy of the resolvent operator. Spectral radius formula.

Course Outcomes

Upon successful completion of the course, the student shall be able to:

- Understand some new concepts in normed spaces and Banach spaces and their main properties.
- Differentiate between finite and infinite dimensional normed spaces.
- Understand the idea of linear operators and linear functionals on normed spaces and relate them to their duals.
- Understand the structure of inner product spaces and differentiate between different kinds of linear operators on Hilbert spaces, self-adjoint, normed and unitary operators.

MATH 613 - Complex Analysis I **(3 credit hrs.)**

Conformality (arcs and closed curves, analytic functions in regions, conformal mapping). Linear Transformations (the linear group, the cross ratio, symmetry, oriented circles, families of circles). Fundamental Theorems (line integrals, rectifiable arcs, line integrals as functions of arcs, Cauchy's Theorem for a rectangle, and Cauchy’s Theorem in a circular disk). Cauchy's Integral Formula (the index of a point with respect to a closed curve, the integral formula, higher derivatives). Local Properties of Analytic Functions (removable singularities, Taylor's Theorem, zeros and poles, the local mapping, the maximum principle). The General Form of Cauchy's Theorem (chains and cycles, simple connectivity, exact differentials in simply connected regions, multiply connected regions). The Calculus of Residues (the residue theorem, the argument principle, evaluation of definite integrals). Harmonic Functions (definition and basic properties, the Mean-value property, Poisson's Formula, Schwarz's Theorem).

Course Outcomes

Upon successful completion of the course, the student shall be able to:

- Prove certain fundamental theorems about analytic functions, e.g. Cauchy integral formula including homotopic version.
- Determine and understand of a deeper aspects of complex variables such as Riemann mapping theorem.

- Use Taylor and Laurent series expansions to derive properties of analytic and meromorphic functions.
- Apply methods of complex analysis to evaluate definite real integrals.
- Explain the theory of analytic functions and prove most important theorems.
- Understand the Mobius transforms mapping and some of their main properties.

MATH 616 - Theory of Operators (3 credit hrs.)

Banach algebra, inverse of an element, compact operators, spectral and resolvent compact linear operators in normed spaces. Seperablity of the range and spectral properties of compact linear operators. Operator equations, Fredholm type theorems. Spectral properties of bounded self adjoint operators. Positive operators. Projections. Spectral families. Special representations of bounded self adjoint operators. Unbounded linear operators and their Hilbert adjoint operators. Symmetric and self adjoint operators. Closed linear operators. Closable operators and their closures. Multiplicand operators and differentiation operators. Polar decomposition. Square root of an operator, partial isometries.

Course Outcomes

Upon successful completion of the course, the student shall be able to:

- Understand the theory of linear operators and differentiate between different kinds of operators.
- Compute the resolvent set, spectral and spectral radius of some kinds of compact operators.
- Understand deeply the spectral theory of linear operators and prove certain properties.
- Understand and analyze some known theorems, as Fredholm type theorem.
- Understand the theory of unbounded operators, closed operators, closable operators and their main properties.
- Study and analyze special families of operators, positive operators, projections, multiplicand operators and differentiation operators.

MATH 621 – Advanced Numerical Analysis (3 credit hrs.)

Spline interpolation, Numerical solution of nonlinear systems of equations, Numerical solution of partial differential equations, Numerical solution of the eigenvalue problem.

Course Outcomes

Upon successful completion of the course, the student shall be able to:

- Construct linear, quadratic, and cubic splines.
- Derive and apply finite differences methods and finite element methods.
- Know the different numerical methods that solve the eigenvalue problem.
- Learn the concept of fixed point for systems of nonlinear equations including existence and uniqueness conditions.
- Learn various iterative methods for approximating solutions to systems of nonlinear equations and convergence behavior.

MATH 623 - Approximation Theory (3 credit hrs.)

The general interpolation problem, Polynomial and Trigonometric interpolation, Best approximation, Best approximation in inner product spaces, Projections, best approximation in the maximum norm, Iterative methods for nonlinear equations, Approximation in several variables.

Course Outcomes

Upon successful completion of the course, the student shall be able to:

- Understand the interpolation problem in its general context
- Find and characterize the best approximation in normed and inner product spaces.
- Understand the concept of projections and their properties in Hilbert space settings.
- Find the best approximation in the Maximum norm.
- Learn various iterative methods for approximating solutions to nonlinear equations.
- Grasp several approximating methods for functions of several variable using polynomials.

MATH 641 – Modern Algebra I **(3 credit hrs.)**

Group actions. Sylow theorems. Groups of automorphisims. The fundamental theorem of finitely generated abelian groups. Classification of groups of small orders. Rings. Homomorphisms of rings. Euclidean domains, P.I.D's, and UFD's. Module over a ring, Submodules. Homomorphisms of modules. Direct sum of modules. Free modules.

Course Outcomes

Upon successful completion of the course, the student shall be able to:

- Deal with the important properties of general groups.
- Understand the action of a group on a set.
- Do calculations concerning the Sylow Theorems.
- Understand the Fundamental Theorem of Finite Abelian groups.
- Understand the concepts of Solvable and Nilpotent groups;
- Deal with some important types of rings: Principal ideal Domains, Euclidean Domains and Unique factorization Domains.
- Grasp the concept of a module over a ring.

MATH 642 – Modern Algebra II **(3 credit hrs.)**

Simple groups and simplicity of A_n. Normal series and solvable groups. Field extensions: Algebraic and transcendental extensions, Algebraic closure. Separable extensions. Normal extensions and normal closure. Splitting fields. Field automorphisims. The Galois group of an extension. The Galois group of a polynomial. The fundamental theorem of Galois theory. Applications: Solvability by radicals, ruler and compass constructions, Finite fields. Purely transcendental extensions. Luroth's theorem.

Course Outcomes

Upon successful completion of the course, the student shall be able to:

- Have a good understanding of field extensions.
- Calculate field automorphisims of certain fields extension.
- Have a good understanding of the fundamental theorem of Galois Theory.
- Deal with some applications of Galois Theory.
- Demonstrate and grasp the basics of finite fields.

MATH 643 – Modern Algebra III **(3 credit hrs.)**

Noetherian rings and modules, Artinian rings and modules, Hilbert's basis theorem for polynomial rings for power series rings, Primary decomposition, Nakayama's lemma, Localization, Integral extensions of rings, Algebraic sets, Hilbert's Nullstellensatz, Noether's normalization theorem, Radicals, Semi-simple rings, Group rings and Masche's theorem, Wedderburn-Artin theorem, Modules, Homomorphism of modules.

Course Outcomes

Upon successful completion of the course, the student shall be able to:

- Deal with the concepts of Noetherian (Artinian) rings and modules, primary decomposition, localization, radicals, polynomial rings, integral extensions of rings, algebraic sets, semi-simple rings, group rings and homomorphism of modules.
- Apply the Hilbert's theorem, the Nakayama lemma, the Masche's theorem and the Wedderburn-Artin theorem.

MATH 648 - Advanced Linear Algebra (3 credit hrs.)

Vector spaces, Dual spaces, Linear transformations and matrices, Invariant subspaces,
Characteristic polynomial, Hamilton-Calay's theorem, Jordan normal form, Diagonalizability, Bilinear and quadratic functions, Lagrange algorithm; Sylvester
Criterion, Euclidean and Unitary spaces, Orthogonal and unitary transformations; Self adjoint transformations, tensor products, Canonical Isomorphisms of tensor products
Tensor algebra of vector spaces.

Course Outcomes

Upon successful completion of the course, the student shall be able to:

- Calculate the invariant subspaces of a linear transformation.
- Calculate the canonical forms of a linear transformation.
- Orthogonally diagonalize symmetric matrices.
- Understand the concepts of dual spaces, bilinear and quadratic functions, orthogonal and unitary transformations.
- Use the concepts covered in this course to grasp theoretical results and concepts of other areas of algebra.

MATH 649 –Advanced Number Theory and Cryptography (3 credit hrs.)

Divisibility in integral domains, Euclidean algorithm, Congruences, Finite fields, Quadratic residues and reciprocity law, Some simple cryptosystems, Enciphering matrices, Public key cryptography, RSA, Discrete log, Knapsack, Pseudoprims, The rho method, Fermat factorization, Continued fractions methods, Elliptic curves.

Course Outcomes

Upon successful completion of the course, the student shall be able to:

- Understand some factoring methods such as pollard rho and Pollard p -1.
- Understand the concepts of quadratic residues and reciprocity law.
- Perform encryptions and decryptions.
- Understand the importance of elliptic curves in cryptography, factoring and primality testing.

MATH 652 - (Fuzzy Set Theory and its Applications) (3 credit hrs.)

Fuzzy Sets, Constructing Fuzzy Sets, Operations on Fuzzy Sets, Decomposition, Theorems, Extension Principle, Fuzzy Numbers. Fuzzy Arithmetic, Possibility Theory, Fuzzification in Integration, Applications in Operations Research.

Course Outcomes

Upon successful completion of the course, the student shall be able to:

- Construct Fuzzy Sets.
- Understand the operations on fuzzy sets.

- Understand the Decomposition Theorem.
- Understand Extension Principle of fuzzy sets.
- Understand Fuzzy Numbers.
- Understand Fuzzy Arithmetic.
- Understand the Possibility Theory.
- Understand Fuzzifications in Integrations.
- Apply the concepts of fuzzy sets in Operations Reasearch.

MATH 661 - Advanced General Topology I **(3 credit hrs.)**

A quick revision of the basic concepts, Neighborhood systems, General product spaces, Tychonoff topology, box topology, Quotient topology and identification spaces, sequences and convergence in first countable spaces, inadequacy of sequences, Nets and filters. More on separation axioms, Jone's Lemma, Urysohn's Lemma, Tietze theorem. More on countability axioms. Covering properties, compact spaces, countably compact spaces, sequentially compact spaces, Lindeloff spaces, local compact spaces, paracompact spaces. Metric spaces, product of metrizable spaces, complete metric spaces and completions, the Baire theorem.

Course Outcomes

Upon successful completion of the course, the student shall be able to:

- Have good skills in metric spaces,
- Have good skills in compactness, locally compactness,
- Paracompactness
- Apply definitions and theorems in finding example and counter examples for several types of spaces.
- Have the ability to deal with the product spaces.
- Apply topological, hereditary and productive properties to solve some problems
- Determine characterizations of certain spaces
- Have the ability to distinguish between locally finite, point finite, discrete collections.

MATH 662 - Advanced General Topology II **(3 credit hrs.)**

Compactifications, the Stone-Cech compactification. Metrizabillity, Urysohn's metrization theorem, Full normality and Stone's coincidence theorem, Alexandroff-Urysohn metrization theorem, Nagata smirnov metrization theorem, Bing's metrization theorem of Moore spaces, Moore metrization theorem. Uniform spaces, Uniform topology, Uniform covers, Operations on Uniform spaces, Uniform continuity, Uniformization, Metrizabillity of Uniform spaces, Totally bounded and complete Uniform spaces, completion. Function spaces, pointwise convergence, Uniform convergence, Uniform metric, compact open topology, Equicontinuity and compactness of spaces of functions, The stone-Weierstrass theorem.

Course Outcomes

Upon successful completion of the course, the student shall be able to:

- Have good skills in metrizable spaces.
- Apply definitions and theorems in finding the relationships between spaces.
- Have the ability to deal with new topologies.
- Determine characterizations of certain spaces.
- Finding the relationships between certain spaces.
- Finding different types of topologies.
- Shown the ability of working independently and with groups.

MATH 663-Algebraic Topology I **(3 credit hrs.)**

Homotopy theory, Fundamental group, Covering spaces, the fundamental group of some surfaces, Applications. Singular homology, the homotopy axiom, the Hurewicz theorem, Exact homology sequences, the singular homology group, Reduced homology, Mayer-Vietoris sequences, Homology of some surfaces, Simplicial complexes, CW-complexes.

Course Outcomes

Upon successful completion of the course, the student shall be able to:

- Define homotopy, path homotopy, punctured plane, fundamental group, first homotopy group.
- Calculate the fundamental group of several surfaces and carves with some applications.
- Introduce the singular homology.
- Understand the axiom of homotopy.
- Find the exact homology sequences and reduced homology of the surfaces.
- Apply Mayer-Vietoris sequences.
- Introduce CW-complexes.

MATH 671 - Advanced Mathematical Methods I **(3 credit hrs.)**

Introduction to integral equations, classification of integral equations, integral transformations, some applications of integral equations, Voltera and Fredholm integral equations of the first and second kind, techniques to solve Voltera and Fredholm integral equations of first and second kind, theoretical results about the existence and uniqueness of the solution of the integral equations.

Course Outcomes

Upon successful completion of the course, the student shall be able to:

- Understand Voltera integral equations of the first and second kind, and to let him know how to solve them.
- Understand Fredholm integral equations of the first and second kind, and to let him know how to solve them.
- Modeling some real problems into integral equations.
- Prove some Theorems related to the existence of solutions of an integral equation.

MATH 676 - Applied Graph Theory **(3 credit hrs.)**

This course aims to introduce students to a variety of different concepts of graph theory, it includes: graphs, connected graphs, some special classes of graph such as (paths, cycles, wheels, regular, complete, partite, …, etc.), degrees of vertices, degree sequences, adjacency and incident matrices, isomorphic graphs, trees, connectivity, Menger’s Theorem, Eulerian and Hamiltonian graphs, planarity, dual graphs, coloring of vertices(edges, maps), Operations on graphs such as union, addition, and some types of products as (Cartesian, direct, strong), Ramsey numbers, and extremal graphs and Turan’s Theorem.

Course Outcomes

Upon successful completion of the course, the student shall be able to:

- Understanding of the basic definitions, concepts, and theorems in graph theory.
- Classify graphs according to their classes, such as connected, planar, non-planar, Eulerian, Hamiltonian … etc.
- Master finding degree sequences, adjacency and incidence matrices, connectivity, vertex (edge, map) colorability, and dual graphs.
- Find Ramsey numbers for some graphs.
- Provide solutions (proofs) of some simple and complex problems (Theorems) in graph theory.

MATH 697 - Selected Topics in Mathematics (3 credit hrs.)

Certain mathematics subjects chosen by the instructor and not to be part of the contents of the courses offered by the Department.

Course Outcomes

It depends on the subjects offered by the instructor.

MATH 699A – Thesis (0 credit hrs.)

MATH 699B – Thesis (3 credit hrs.)

MATH 699C – Thesis (6 credit hrs.)

MATH 699D – Thesis (9 credit hrs.)